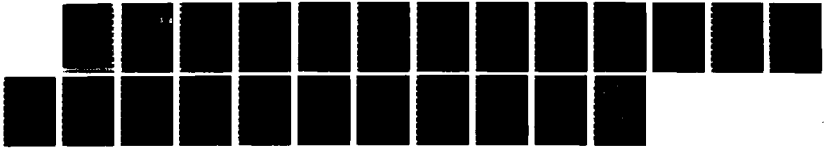
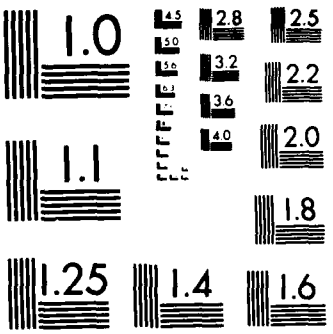
AD-A174 919 THE STUDY OF CERTAIN ASPECTS OF PROBABILITY WITH 1/1
APPLICATIONS IN COMMUNIC (U) TEXAS UNIV AT AUSTIN DEPT
OF ELECTRICAL AND COMPUTER ENGINEER G L WISE
UNCLASSIFIED 30 JUN 86 AFOSR-TR-86-2212 AFOSR-81-0047 F/G 17/2 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

AD-A174 919

DTIC FILE COPY

AFOSR-TR- 86-2212

DTIC ELECTE DEC 10 1986 S D

FINAL TECHNICAL REPORT

GRANT AFOSR-81-0047

Grant Period: October 1, 1980 - September 30, 1985

Approved for public release; distribution unlimited.

Gary L. Wise

Department of Electrical and Computer Engineering

and Department of Mathematics

University of Texas at Austin

Austin, Texas 78712

(512) 471-3356

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL
This technical report has been reviewed and is
approved for public release IAW AFR 190-12.
Distribution is unlimited.
MATTHEW J. KELLY
Chief, Technical Information Division

86 12 09 049

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	AFOSR-81-0047

8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS.			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
Bldg. 410 Bolling AFB, DC 20332-6448	6.1102F	2304	A5	

11. TITLE (Include Security Classification) The Study of Certain Aspects of Probability with Applications to Communication Theory

12. PERSONAL AUTHOR(S)
Gary L. Wise

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
final	FROM 10/1/80 TO 9/30/85	86 June 30	21

16. SUPPLEMENTARY NOTATION
Final Technical Report

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	quantization, signal detection; estimation

INTRODUCTION

Many aspects of signal processing and communication theory have proven to be limited by a lack of sufficient developments in the areas of probability theory and mathematical statistics. Our investigations attempted to overcome this deficiency by contributing both to the underlying theoretical basis of the area as well as to some applied aspects of the area, and we have obtained a large body of results. Some principal thrust areas of our research effort have been concerned with quantization theory, signal detection, and estimation theory.

A brief topical overview of our research areas follows.

Quantization -
- existence of optimal quantizers,
- convergence properties of sequences of quantizers,
- design of quantizers;

Signal detection -
- effects of statistical dependence,
- relative efficiency between detectors,
- nonparametric detection;

Estimation -
- effects of imperfect data,
- a general class of fidelity criteria leading to nonlinear estimators,
- convergence properties of sequences of estimators;

Additional areas -
- median filters,
- stability of differential equations with random coefficients;
- bandwidth properties of random processes;
- contention resolution in local area computer networks;
- image compression;
- counterexamples.

This report is a survey of the technical results ensuing from the Grant AFOSR-81-0047. In the next section, we list the individuals supported by the grant. Then we list the publications which were supported by this grant. We conclude with a brief survey of the research results.

PERSONNEL

Principal Investigator:

Gary L. Wise

Post-doctoral Research Associates:

Efren F. Abaya

Aristotle Arapostathis

Don R. Halverson

Federico Kuhlmann

Michael P. Starbird

Research Assistants:

Efren F. Abaya (received his Ph.D. degree in August 1982. His dissertation was entitled "A Theoretical Study of Optimal Vector Quantizers.")

Nahid Khazenie

Federico Kuhlmann (received his Ph.D. degree in May 1981. His dissertation was entitled "Analysis and Design Considerations for Two Nonlinear Systems in Communication Theory: Median Filters and Nonuniform Quantizers.")

Yih-Chiao Liu (received his Ph.D. degree in December 1985. His dissertation was entitled "An Investigation of Some Current Problems in Data Communication Theory.")

Fu-Sheng Lu (received his Ph.D. in May 1984. His dissertation was entitled "Efficient Methods for the Design of Scalar Quantizers.")

John M. Morrison (anticipates receiving his Ph.D. in August 1986. His dissertation is tentatively entitled "Some Problems in Martingale-Like Operators and Flows of σ-Algebras Arising in Communication Theory.)

Senior Secretaries:

Barbara Calaway

Deborah Dierdorf

Cindy Torrance

Joan Van Cleave

Accesion For
NTIS CRA&I ☑
DTIC TAB ☐
Unannounced ☐
Justification .....
By .....
Distribution /
Availability Codes
Dist Special
A-1

PUBLICATIONS SUPPORTED BY GRANT AFOSR-81-0047

1. D. R. Halverson and G. L. Wise, "On the performance of approximately optimal memoryless detectors for signals in dependent noise," PROCEEDINGS OF THE 1981 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 25-27, 1981, pp. 147-152.

2. F. Kuhlmann, J. A. Bucklew, and G. L. Wise, "Nonuniform quantization and transmission of generalized Gaussian signals over noisy channels," PROCEEDINGS OF THE 1981 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 25-27, 1981, pp. 345-350.

3. E. F. Abaya and G. L. Wise, "Some notes on optimal quantization," PROCEEDINGS OF THE 1981 INTERNATIONAL CONFERENCE ON COMMUNICATIONS, Denver, Colorado, June 14-18, 1981, pp. 30.7.1-30.7.5.

4. F. Kuhlmann and G. L. Wise, "Piecewise linear companders for nonuniform quantization," PROCEEDINGS OF THE 24TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Albuquerque, New Mexico, June 29-30, 1981, pp. 168-172.

5. D. R. Halverson, G. L. Wise, and H. V. Poor, "On an aspect of linear correction terms for nonlinear detection systems," PROCEEDINGS OF THE 24TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Albuquerque, New Mexico, June 29-30, 1981, pp. 356-360.

6. G. L. Wise, "Some results on the stability of systems with multiplicative state noise," PROCEEDINGS OF THE 24TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Albuquerque, New Mexico, June 29-30, 1981, pp. 361-364.

7. D. R. Halverson and G. L. Wise, "Asymptotic memoryless detection of random signals in dependent noise," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 312, pp. 13-29, July 1981.

8. G. L. Wise and N. C. Gallagher, "A novel approach for the computation of orthogonal polynomial expansions," JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS, Vol. 7, pp. 157-160, September 1981.

9. F. Kuhlmann and G. L. Wise, "On second moment properties of median filtered sequences of independent data," IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-29, pp. 1374-1379, September 1981.

10. E. F. Abaya and G. L. Wise, "Convergence of vector quantizers with application to the design of optimal quantizers," PROCEEDINGS OF THE NINETEENTH ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, September 30-October 2, 1981, pp. 79-88.

11. D. R. Halverson and G. L. Wise, "An observation concerning signal to noise ratio properties of continuous and discrete time detectors," PROCEEDINGS OF THE NINTEENTH ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, September 30-October 2, 1981, pp. 569-575.

12. F. Kuhlmann and G. L. Wise, "Design considerations for asymptotically optimal piecewise linear companders," PROCEEDINGS OF THE 1981 NATIONAL TELECOMMUNITIONS CONFERENCE, New Orleans, Louisiana, November 29-December 3, 1981, pp. F4.4.1-F4.4.4.

13. N. C. Gallagher and G. L. Wise, "A theoretical analysis of the properties of median filters," IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, Vol. ASSP-29, pp. 1136-1141, December 1981.

14. F. Kuhlmann and G. L. Wise, "The bivariate distribution of a median smoothed Markov chain," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 313, pp. 107-118, February 1982.

15. J. A. Bucklew and G. L. Wise, "Multidimensional asymptotic quantization theory with r-th power distortion measures," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-28, pp. 239-247, March 1982.

16. T. E. McCannon, N. C. Gallagher, D. Minoo-Hamedani, and G. L. Wise, "On the design of nonlinear discrete time predictors," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-28, pp. 366-371, March 1982.

17. D. R. Halverson and G. L. Wise, "On the performance of a nonparametric detection scheme for dependent data," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-28, pp. 380-384, March 1982.

18. D. L. Michalsky, G. L. Wise, and H. V. Poor, "A relative efficiency study of some popular detectors," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 313, pp. 135-148, March 1982.

19. D. R. Halverson, N. C. Griswold, and G. L. Wise, "On generalized block truncation coding quantizers for image compression," PROCEEDINGS OF THE 1982 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 17-19, 1982, pp. 47-52.

20. E. F. Abaya and G. L. Wise, "Some remarks on optimal quantization," PROCEEDINGS OF THE 1982 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 17-19, 1982, pp. 60-65.

21. G. L. Wise, "A result on the stability of linear differential equations with random coefficients," SYSTEMS AND CONTROL LETTERS, Vol. 1, pp. 385-389, May 1982.

22. F. Kuhlmann and G. L. Wise, "Performance of median filters with random inputs," PROCEEDINGS OF THE 1982 INTERNATIONAL CONFERENCE ON COMMUNICATIONS, Philadelphia, Pennsylvania, June 13-17, 1982, pp. 1H.2.1-1H.2.5.

23. D. R. Halverson and G. L. Wise, "A further comment on the spectral width of MSK-type signaling waveforms," IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-30, pp. 1983-1984, August 1982.

24. E. F. Abaya and G. L. Wise, "Quantization with non-difference based distortion measures," PROCEEDINGS OF THE TWENTIETH ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 6-8, 1982, pp. 281-289.

25. G. L. Wise, " Effects of a zero memory nonlinearity on the spectrum of a random process," PROCEEDINGS OF THE TWENTIETH ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 6-8, 1982, pp. 366-373.

26. E. F. Abaya and G. L. Wise, "On the existence of optimal quantizers," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-28, pp. 937-940, November 1982.

27. F. Kuhlmann, J. A. Bucklew, and G. L. Wise, "Compressors for combined source and channel coding with applications to the generalized Gaussian family," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-29, pp. 140-144, January 1983.

28. D. R. Halverson, G. L. Wise, and M. W. Thompson, "Robust detection of a constant signal in Laplace noise," PROCEEDINGS OF THE 1983 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 23-25, 1983, pp. 228-232.

29. E. F. Abaya and G. L. Wise, "Some aspects of quantization in metric spaces," PROCEEDINGS OF THE 1983 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 23-25, 1983, pp. 776-780.

30. D. R. Halverson and G. L. Wise, "On an aspect of matched filters in continuous and discrete time," CIRCUITS, SYSTEMS, AND SIGNAL PROCESSING, Vol. 2, pp. 269-276, 1983.

31. Y.-C. Liu and G. L. Wise, "Some relative efficiency results for detectors for time varying signals," PROCEEDINGS OF THE 1983 INTERNATIONAL CONFERENCE ON COMMUNICATIONS, Boston, Massachusetts, June 19-22, 1983, pp. 1592-1596.

32. G. L. Wise, "Some common misconceptions in communication theory," PROCEEDINGS OF THE 26TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Puebla, Mexico, August 15-16, 1983, pp. 33-37.

33. F.-S. Lu and G.L. Wise, "A further investigation of the Lloyd-Max Algorithm for Quantizer Design," PROCEEDINGS OF THE TWENTY-FIRST ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 5-7, 1983, pp. 481-490.

34. D.R. Halverson and G.L. Wise, "Signal detection when dependence is neglected," PROCEEDINGS OF THE TWENTY-FIRST ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 5-7, 1983, pp. 622-630.

35. E. F. Abaya and G. L. Wise, "Convergence of vector quantizers with applications to optimal quantization," SIAM JOURNAL ON APPLIED MATHEMATICS, Vol. 44, pp. 183-189, February 1984.

36. D. R. Halverson and G. L. Wise, "Asymptotic memoryless discrete-time detection of (phi)-mixing signals in (phi)-mixing noise," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-30, pp. 415-417, March 1984.

37. D. R. Halverson and G. L. Wise, "Approximately optimal memoryless detection of random signals in dependent noise," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-30, pp. 420-424, March 1984.

38. D. R. Halverson and G. L. Wise, "On the approximation of nonlinearities for memoryless detection," PROCEEDINGS OF THE 1984 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 14-16, 1984, pp. 302-305.

39. F.-S. Lu and G. L. Wise, "A modified companding method for quantization," PROCEEDINGS OF THE 1984 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Princeton, New Jersey, March 14-16, 1984, pp. 577-580.

40. F.-S. Lu and G. L. Wise, "A simple approximation for minimum mean squared error symmetric uniform quantization," IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-32, pp. 470-474, April 1984.

41. D. R. Halverson, N. C. Griswold, and G. L. Wise, "A generalized block truncation coding algorithm for image compression," IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, Vol. ASSP-32, pp. 664-668, June 1984.

42. Y.-C. Liu and G. L. Wise, "On the relative efficiency between the correlation detector and the sign detector," PROCEEDINGS OF THE 27TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Morgantown, West Virginia, June 11-12, 1984, pp. 222- 225.

43. F.-S. Lu and G. L. Wise, "An efficient method for the design of minimum mean absolute error quantizers," PROCEEDINGS OF THE 27TH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Morgantown, West Virginia, June 11-12, 1984, pp. 369-372.

44. D. R. Halverson and G. L. Wise, "On the optimality of data processors for signal detection over a class of contaminated noises," PROCEEDINGS OF THE TWENTY-SECOND ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 3-5, 1984, pp. 328-332.

45. E. F. Abaya and G. L. Wise, "Some comments on the design of quantizers," PROCEEDINGS OF THE TWENTY-SECOND ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 3-5, 1984, pp. 579-586.

46. D. R. Halverson and G. L. Wise, "A result on neglecting dependence in signal detection," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-30, pp. 856-859, November 1984.

47. E. F. Abaya and G. L. Wise, "Some remarks on the existence of optimal quantizers," STATISTICS AND PROBABILITY LETTERS, Vol. 2, pp. 349-351, December 1984.

48. N. C. Griswold, D. R. Halverson, and G. L. Wise, "A result on data compression for image processing," PROCEEDINGS OF THE 1985 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 27-29, 1985, pp. 172-176.

49. J. M. Morrison and G. L. Wise, "A stability property of conditional expectations," PROCEEDINGS OF THE 1985 CONFERENCE ON INFORMATION SCIENCES AND SYSTEMS, Baltimore, Maryland, March 27-29, 1985, pp. 226-229.

50. G. L. Wise, "A note on a common misconception in estimation," SYSTEMS AND CONTROL LETTERS, Vol. 5, pp. 355-356, April 1985.

51. D. R. Halverson and G. L. Wise, "On the performance of a nonparametric detection scheme for strong mixing noise," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-31, pp. 522-527, July 1985.

52. Y.-C. Liu and G. L. Wise, "A note on estimation with quantized data," IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING, Vol. ASSP-33, pp. 1619-1621, December 1985.

53. F.-S. Lu and G. L. Wise, "A further investigation of Max's algorithm for optimum quantization," IEEE TRANSACTIONS ON COMMUNICATIONS, Vol. COM-33, pp. 746-750, July 1985.

54. Y.-C. Liu and G. L. Wise, "A performance analysis of a CSMA/CD protocol," PROCEEDINGS OF THE TWENTY-EIGHTH MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS, Louisville, Kentucky, August 19-20, 1985, pp. 241-244.

55. J. M. Morrison and G. L. Wise, "A well-posedness property of a class of variational problems and its application to nonlinear estimation," PROCEEDINGS OF THE TWENTY-THIRD ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 2-4, 1985, pp. 28-34.

56. Y.-C. Liu and G. L. Wise, "An analysis of a CSMA/CD collision resolution scheme," PROCEEDINGS OF THE TWENTY-THIRD ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL, AND COMPUTING, Monticello, Illinois, October 2-4, 1985, pp. 533-542.

57. D. R. Halverson and G. L. Wise, "On nonlinearities in asymptotic memoryless detection," IEEE TRANSACTIONS ON INFORMATION THEORY, Vol. IT-32, pp. 292-296, March 1986.

58. J. M. Morrison and G. L. Wise, "An asymptotic property of nonlinear estimators arising as solutions to a certain class of convex programming problems," to appear in SYSTEMS AND CONTROL LETTERS, Vol. 7, 1986.

59. E. F. Abaya and G. L. Wise, "Convergence of vector quantizers with non-difference based distortion measures," to appear in JOURNAL OF MULTIVARIATE ANALYSIS.

A SURVEY OF RESULTS

In this section we will present a brief survey of our results supported by the grant. Much of our effort was concerned with quantization theory, signal detection, and estimation theory. Additional effort was directed to several other areas.

Quantization is the process by which data is reduced to a simpler, more coarse representation which is more compatible with digital processing. Loosely speaking, quantization is at the heart of analog to digital conversion. It is an area which has increased in importance in the last few years due to the burgeoning advances in digital technology. The typical goal of quantization is to reduce data to a simpler representation without causing much distortion; that is, the output of a quantizer should be close to the input, with some appropriate measure of distance.

An N-level k-dimensional vector quantizer is a mapping $Q: \mathbb{R}^k \to \mathbb{R}^k$ which assigns the input vector x to an output vector Q(x) chosen from a set of N vectors $\{y_i: y_i \in \mathbb{R}^k, i=1,\ldots,N\}$. Generally, the quantizer input is modeled as a random vector X described by a k-variate distribution F. A measure of quantizer performance is the distortion function

$$D(Q,F) = \int d(x,Q(x))dF(x), \tag{1}$$

where $d: \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}$ is an appropriately chosen cost function. Many traditional cost functions d(x,y) were based on the quantization error x-y; for example, the r-th power cost function $||x-y||^r$ where $||\cdot||$ represents the Euclidean norm on $\mathbb{R}^k$. An optimal N-level quantizer Q for the random vector X is one that minimizes (1) over the class of all N-level quantizers.

It would be of interest to know conditions under which optimal

quantizers exist. For example, in recent years much work has been done to investigate design algorithms for optimal quantization. Existence of an optimal quantizer is directly related to the convergence of such algorithms. If an optimal quantizer does not exist, then the algorithm might not converge, or it may converge to a non-optimal solution. We have investigated numerous aspects of the existence of optimal quantizers. Our results have addressed difference based cost functions, e.g. $d(x,y) = c(x-y)$, as well as non-difference based cost functions for k-dimensional random variables. We have established sufficient conditions for existence of optimal quantizers, and, for a fairly general case, we have established easily checked conditions which are necessary and sufficient for existence of an optimal quantizer. We also investigated the existence of optimal quantizers for random variables taking values in a metric space. Our results on the existence of optimal quantizers are given in [3, 10, 20, 24, 26, 29, 35, 45, 47, and 59].

One of the more practical problems associated with quantizers is the problem of how to construct them. Most of the algorithms for quantizer design involve successively improving a suboptimal quantizer, with the procedure hopefully converging to an optimal quantizer. Assume that a sequence of distribution functions $\{F_n\}$ converges weakly to a distribution function F, and let Q_n be an optimal N-level quantizer for F_n. Consider the following questions:

(Q1) Do the distortions $D(Q_n,F_n)$ converge to $D(Q,F)$, where Q is an optimal quantizer for F?

(Q2) Does $\{Q_n\}$ converge weakly to some quantizer Q; and if so, is Q optimal for F?

Loosely speaking, these two questions formalize the essential concerns in studying the convergence of algorithms for quantizer design. In our

investigations we established conditions sufficient for an affirmative answer to both of these questions. We considered k-dimensional random variables with both difference and non-difference based cost functions, and we also considered random variables taking values in a metric space; in each case we addressed both of these questions and gave conditions implying affirmative answers. Our results on the convergence properties of sequences of quantizers can be found in [10, 15, 20, 24, 29, 35, 45, and 59].

Many researchers have studied algorithms for optimal quantization of a known probability distribution. In practice, however, the statistical description of the signal is often not known precisely. If the input distribution F is not known, then we might form an estimate F_n based on n observations of the input signal. As n becomes large, we expect a reasonable estimate to converge to the true distribution F. Intuitively then an optimal quantizer designed for F_n and the resulting distortion should closely approximate those of an optimal quantizer for F. We established properties of an estimator F_n so that this kind of reasoning would be valid, and we gave some specific results for a kernel density estimate. Our results in this area are given in [10, 20, 24, 29, 35, and 45].

We have established several results concerned with the practical aspect of the actual design of scalar quantizers. For the design of scalar minimum mean square error quantizers, the most popular algorithm is that credited to Lloyd (method II) and Max. Two tactical problems associated with the Lloyd-Max algorithm are how to initialize it and how to update it. We investigated both of these problems; we presented an initialization scheme which was markedly superior to existing schemes, and we presented an update procedure based on the Newton-Raphson algorithm. Our work resulted in a very fast design procedure. In another approach to the design of scalar

quantizers, we considered their implementation via a method known as companding, where a compressor maps the reals into (0,1), followed by a uniform quantizer on (0,1), followed by an expander (the inverse of the compressor). We investigated the use of piecewise linear companders (i.e. the compressor was piecewise linear). We also analyzed the performance of companders in the presence of additive channel noise; the compressor was designed to not only take into account the quantization noise but also to take into account the presence of noise that might be present during the transmission of data. Our results on the actual numerical design of quantizers are in [2, 4, 12, 27, 33, 39, 40, 43, and 53].

Another research area in which we have obtained results is the area of discrete time signal detection. The detection problem is modeled as a test between two statistical hypotheses; we assume that under the null hypothesis noise alone is being observed, and under the alternate hypothesis a signal plus noise is being observed. In the case of discrete time detection where the noise and the signal are stationary and the samples are mutually independent, it is well known that the Neyman-Pearson test has a test statistic which can be expressed as $\sum_{i=1}^{n} g(X_i)$ where X_i, i=1,...,n, represent the observations, and $g(\cdot)$ is an appropriately chosen function. We considered the problem of constraining the test statistic to be of the above form and letting the noise samples be "slightly" dependent. We then tried to choose the function $g(\cdot)$ to best account for the dependency structure of the noise, in the sense of the asymptotic relative efficiency (or Pitman efficiency) with respect to any other choice for $g(\cdot)$. We considered various ways of formalizing the concept of a sequence of random variables being "slightly" dependent, including the concepts of ϕ-mixing and strong mixing, as well as concepts based upon the maximal correlation function, and we

treated both constant signals and random signals. Our work in this area can be found in [1, 5, 7, 36, 37, 38, and 53].

We also studied signal detection based upon a finite number of observations. Consider the case where one simply assumes that the observations $\{X_i, i=1,2,\ldots,n\}$ form a set of mutually independent random variables. Under this assumption (and knowing all of the univariate distributions) one could straightforwardly design a detector optimal in the Neyman-Pearson sense. Assume that it is designed for a false alarm probability of α and that (under the assumption of independence) the detection probability is β. One might suspect that the data is not really mutually independent but that any dependency present is negligible. What if there is dependency in the data (as there surely would be with today's high sampling rates)? In that case, if the above detector is used, the actual false alarm rate would be $\tilde{\alpha}$ and the actual detection probability would be $\tilde{\beta}$. In our work we related a measure of dependency to bounds on $|\alpha-\tilde{\alpha}|$ and $|\beta-\tilde{\beta}|$. Our results in this area can be found in [34 and 46].

It is well known that for a wide variety of fidelity criteria (Neyman-Pearson, Bayes, probability of error), an optimal detection scheme consists of comparing a likelihood ratio (i.e. an appropriate Radon-Nikodym derivative) to an appropriate threshold. In a detection context, the likelihood ratio is often thought of as the data processor. We considered the effect induced on the data processor of a signal detection system when the underlying noise distribution is varied about its nominal value, and we addressed the concern of how a noise distribution can be changed without the optimal data processor being changed. Our results characterized a class of contaminants of an arbitrary nominal distribution over which the optimal data processor can be designed using the nominal distribution. This work can be

found in [44].

In another aspect of signal detection, we considered using a continuous time filter for a discrete time problem. That is, we supposed that the signal $s(\cdot)$ was known at certain discrete instants of time $\{t_k\}$, and we showed that under certain regularity conditions we could derive a signal $\tilde{s}(\cdot)$ such that $\tilde{s}(t_k) = s(t_k)$ in such a way that if we used a continuous time filter based on the signal $\tilde{s}(t)$, $0 \leq t \leq T$, then the performance of the detector was robust to the inexact knowledge of the actual signal $s(t)$, $0 \leq t \leq T$. We showed that the performance of the continuous time detector based on the signal $\tilde{s}(t)$, $0 \leq t \leq T$, could be considerably better than the performance of the discrete time detector based on the signal $\{s(t_k)\}$, even though the signal $s(t)$ is not known for t outside the set $\{t_k\}$. This work can be found in [11 and 30].

Consider the nonparametric detection of a constant signal in noise. If the noise sequence were independent and identically distributed, a popular nonparametric detector might be the sign detector. Other investigators considered the case where the noise instead was m-dependent, and they then considered a modified sign detector. That is, they summed n consecutive samples, skipped the next m, summed the next n, skipped the next m, and so on. The sums thus form a sequence of mutually independent random variables. To this resulting sequence of random variables, they applied the classical sign detector. We considered the use of this sample and skip procedure for this problem of m-dependent noise and also for the problem of strong mixing noise, and we showed how a modified sign detector can be designed. We established an upper bound on the asymptotic performance of the detector, and we specified the form of a detector whose performance achieves this bound. The optimal design of the detector under a finite sample criterion was also

considered, and we showed that, contrary to remarks in the literature, there is a marked difference in the detector designs resulting from the asymptotic criterion and from the finite sample criterion. We produced examples of pairs of nonparametric detectors (D_1, D_2) for which D_1 is "better" and D_2 is "worse" as judged from the asymptotic viewpoint but with the opposite conclusion when judged from the finite sample viewpoint. Our work in this area can be found in [17 and 51].

The relative efficiency between two detectors is a ratio of the amount of data required by one detector, relative to another, to attain a prescribed level of performance. Although this concept is of fundamental importance in the theory of signal detection, it has been successfully investigated in only very few special cases. As an approximation to the relative efficiency, engineers have frequently employed the asymptotic relative efficiency (ARE), the limiting value of the relative efficiency (under suitable regularity conditions) as the sample sizes required by the detectors approach infinity. The ARE was introduced in the statistical literature, where it is known as the Pitman efficiency. Usually it can be determined in a fairly straightforward fashion, and this is due principally to an appeal to the central limit theorem. The ARE is a limiting result, and in any practical engineering situation, only a finite number of samples can be taken in the context of discrete time detection. Thus it might not always be appropriate to approximate the relative efficiency with the ARE. We considered the discrete time detection of signals in additive noise, and we studied how the relative efficiency converged to the ARE. We also considered this situation with signals which were not precisely known, and we presented both upper and lower bounds on the relative efficiency. Some examples illustrated a fairly slow approach of the relative efficiency to the ARE. Our work in this area

can be found in [18, 31, and 42].

Another research area in which we have achieved some results is concerned with estimating a random variable by a function of other random variables. Let $\{X(t),t\in[0,T]\}$ denote a random process that is continuous in probability and let Y denote a second order random variable. In many situations, one is interested in $E(Y|X(t),t\in[0,T])$. This is the optimal $\sigma(X(t),t\in[0,T])$-measurable mean square estimate of Y given perfect knowledge of $\{X(t),t\in[0,T]\}$ at all times $t\in[0,T]$. However, in practical situations we are neither able to observe the random process continuously in time nor do we have perfect knowledge about the random process when we are able to observe it. That is, we can practically observe X(t) at only finitely many times $t_1,\ldots,t_n$, and each observation is actually quantized, since computers and practical measuring devices can handle only finite data sets. We are thus led to consider questions such as how does $E(Y|X(t),t\in[0,T])$ compare to $E(Y|Q[X(t_i)],i=1,\ldots,n)$? We investigated this situation, where the random process $\{X(t),t\in[0,T]\}$ took values in a separable metric space. Since this random process represents potentially observable data, the metric space setting thus allows a more realistic model of a setting in which we might observe data. Also, with appropriate integrability conditions put upon Y, we investigated the estimation of Y given $\{X(t),t\in[0,T]\}$ versus the estimation of Y given our "defective" data $\{Q[X(t_i)],i=1,\ldots,n\}$, for fidelity criteria that are far more general than mean square error. We developed some approximation results for abstract Banach spaces. We then considered Orlicz spaces of random variables and used our Banach space results to establish several convergence properties of estimators based on "defective" data such as $\{Q[X(t_i)], i=1,\ldots,n\}$ to estimators based on ideal data such as $\{X(t), t\in[0,T]\}$. This allowed us to examine estimation theory in a more

general context than mean square error and it gave us a much wider choice of criteria for analyzing and penalizing error. Our results in this area can be found in [49, 55, and 58]. In [52] we investigated the related problem of how $E\{(Y-E(Y|X))^2\}$ differs from $E\{(Y-E(Y|Q(X))^2\}$, where Q is a quantizer.

We have also achieved results in the area of median filtering, a popular nonlinear filtering scheme used in discrete time signal processing. Linear filtering is not adequate for certain applications. If the signal to be processed consists of the sum of a high frequency noise and a component with information-bearing sharp edges, then a linear filter, upon removing the noise, will also smear out the edges in the original signal, possibly obscuring their information contents. To overcome this difficulty, nonlinear smoothing devices have been used. Tukey is generally credited with the idea of introducing nonlinear filters based on moving sample medians of the input signal, and he did the pioneering work in this area. These systems are referred to as median filters. Median filters have several interesting characteristics that make them sometimes superior to linear systems. If a signal has spike-like components, median filtering can eliminate them without significantly modifying other components, and if a signal contains step-like components, median filtering can preserve them. Median filters have been studied by several investigators; however, many of the reported results are of an empirical nature. We presented the first rigorous theoretical analysis of the properties of median filters. Also we analyzed the effect of median filters upon sequences of random variables, and we studied the second moment properties of their outputs. Our work in this area can be found in [9, 13, 14, and 22].

Recently, several investigators have been studying the stability properties of the solutions of differential equations of the following form:

$$\dot{X}(t) = AX(t) + f(t)BX(t),$$

where $X(t) \in \mathbb{R}^n$, $X(0) = X_0$, $f(t)$ is a scalar random process, and A and B are $n \times n$ matrices. Such equations are used to model linear dynamical systems with multiplicative state noise and investigators have been concerned with the conditions under which the state $X(t)$ would asymptotically tend to the origin. Roughly speaking, most investigators have based their analyses upon either certain algebraic properties of the system or upon the assumption of ergodicity of the noise. We presented very general conditions under which the state $X(t)$ tends to the origin with probability one with neither the traditional algebraic assumptions on the system nor ergodic assumptions on the noise. This work can be found in [6 and 21].

In another area, we introduced a novel technique for expanding a function in orthonormal polynomials. Given a function to be expanded in a polynomial series, we first used the FFT (fast Fourier transform) to compute a vector of Fourier coefficients. Then, using a change of basis transformation, we transformed the Fourier coefficients to the polynomial coefficients. This work can be found in [8].

We also considered some aspects of the problem of image compression. In image compression one seeks to reduce the number of bits used to represent an image while at the same time not distorting the image too much by this alteration. Loosely speaking, it is often desirable to employ an image compression scheme whose attractiveness lies not in optimality under a quantitatively defined fidelity criterion, but rather in ease of implementation with good performance. Consider partitioning the image into nonoverlapping blocks of pixels. We investigated a particular algorithm known as block truncation coding, which involves a two level nonparametric quantizer whose output levels are obtained by matching certain sample moments

(of the empirical data in each block) of the data before and after quantization. We generalized the original development by allowing more general sample moments to be preserved by the quantizer; and we showed by examples that our modification could yield better performance. We also introduced an easily implemented alternative scheme for image compression which allows exploiting some commonly encountered image characteristics, such as some relatively large regions of constant gray level. We showed via examples that our scheme can offer remarkably good performance in certain cases for various compression ratios. A noteworthy additional consequence was that in certain cases we could analytically characterize a data compression rate such that the image can be restored precisely (with no distortion whatsoever!). While not intended for application to all images, our method was well suited to offering improved performance and enhanced compression ratios for certain types of images. Our work in this area can be found in [19, 41, and 48].

We have also investigated contention resolution in local area computer networks. Our work has been in the context of carrier sense multiple access with collision detection (CSMA/CD); that is, a station can sense the transmission of another station, and a station can detect a collision. The well-known Ethernet, pioneered by Xerox in 1976, is a popular example of this technique. The basic CSMA procedure is to listen to the channel before transmitting and to defer when the channel is busy. A collision can result due to the propagation delay associated with the network. In 1984 a protocol called Enet II was introduced as a candidate for the second generation of Ethernet, and it was designed to address the problem of effectively resolving collisions in a multiple access protocol. Our investigations have addressed the question of how well this protocol resolves collisions. Our work in this

area can be found in [54 and 56].

In another area of research, we investigated some effects of a form of nonlinear distortion on spectral properties of random processes. Let $\{X(t), t\in\mathbb{R}\}$ denote a second order random process that is second order stationary and mean square continuous. Then it is well known that X(t) possesses a spectral distribution function F. (We take all spectral distribution functions to be zero at minus infinity.) Let $g:\mathbb{R}\to\mathbb{R}$ be a Borel measurable function such that $E\{(g[X(t)])^2\} < \infty$. (The function g is often known as a zero memory nonlinearity.) Then g[X(t)] is mean square continuous and thus possesses a spectral distribution function G. For $p \geq 2$, let

$$A_p = \frac{\int |u|^p \, dF(u)}{F(\infty)}$$

and

$$\overline{A}_p = \frac{\int |u|^p \, dG(u)}{G(\infty)}$$

where we assume that $F(\infty) > 0$ and $G(\infty) > 0$ (i.e. neither X(t) nor $g(\cdot)$ is "degenerate"). The A_p and $\overline{A}_p$ represent the p-th absolute moments of the input and output normalized spectral distributions, respectively. These quantities are important since they relate to measures of spectral dispersion, and thus bandwidth. Bandwidth is a fundamental property of signal transmission; and nonlinearities, such as $g(\cdot)$, are encountered in numerous situations. We compared A_p to $\overline{A}_p$ for various conditions placed on $g(\cdot)$ and X(t), and with certain conditions on X(t) we gave both upper and lower bounds to $\overline{A}_p$ in terms of A_p and $g(\cdot)$. For example, we showed that if X(t) is a zero mean Gaussian process with a nonconstant autocorrelation function and if $g(\cdot)$ is odd and not almost everywhere constant, then $A_2 \leq \overline{A}_2$, with equality if and only if g is almost everywhere linear. Our work in this

area is in [25].

In [16] we presented some results on the prediction of a time series where our predictor was constrained to have the form of a zero memory nonlinearity followed by a linear filter.

In [23] we corrected an error relating to the width of the center spectral lobe of a class of MSK-type (minimum shift keying) signals.

In [28] we presented the preliminary results of an effort we began, concerned with the consequences of robustness in signal detection.

Finally, we presented a collection of counterexamples to some popular misconceptions in communication theory. For example, we showed that for any positive number M, there exists a real number B, a probability space, two random variables X and Y defined on this probability space, and a function $f: \mathbb{R} \to \mathbb{R}$ such that $|X|$ and $|Y|$ are both less than B, $f(Y) = X$ pointwise on the underlying probability space, but $E([Y-E(Y|X)]^2) > M$. These counterexamples are presented in [32] and [50].

END

1-87

DTIC